Newton-Raphson Power Flow Solution

Because of its quadratic convergence, Newton's method is mathematically superior to Gauss-Seidel method and is less prone to divergence with ill-conditioned problems. For large power system, the newton-Raphson method is found to be more efficient and practical. The number of iteration required to obtain a solution is independent of system size, but more functional evaluations are required at each iteration. Since the power flow problem real **power** and **voltage** magnitude are specified for voltage-controlled buses, the power flow equation is formulated in **polar form**.

For the typical bus of the power system shown in Figurer 1-7, the current entering bus ***i*** is given by (1-24).This equation can be rewritten in terms of the bus admittance matrix as

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j$$	(1-48)

In the above equation, **j** includes bus ***i***, Expressing this equation in polar form, we have

$$I_i = \sum_{j=1}^{N} \lvert Y_{ij}\rvert \lvert V_j\rvert \angle\theta_{ij} + \delta_j$$	(1-49)

The complex power at bus ***i*** is

$$P_i - jQ_i = V_i^* I_i$$	(1-50)

Substituting from (1-49) for I_i in (1-50).

$$P_i - jQ_i = \lvert V_i\rvert \angle - \delta_i \sum_{j=1}^{N} \lvert Y_{ij}\rvert \lvert V_j\rvert \angle\theta_{ij} + \delta_j$$	(1-51)

Separating the real and imaginary parts.

$$P_i = \sum_{j=1}^{N} \lvert V_i\rvert \, \lvert V_j\rvert \lvert Y_{ij}\rvert \cos(\theta_{ij} - \delta_i + \delta_j)$$	(1-52)
$$Q_i = -\sum_{j=1}^{N} \lvert V_i\rvert \, \lvert V_j\rvert \lvert Y_{ij}\rvert \sin(\theta_{ij} - \delta_i + \delta_j)$$	(1-53)

Eq. (1-52) and (1-53) constitute a set on nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit, and phase angle in **radians**. We have two equations for each **load bus**, given by (1-52) and (1-53). Expanding (1-52) and (1-53) in Taylor's series about the initial estimate and neglecting all higher order terms results in the following set of linear equations.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_2^{(k)}}{\partial \delta_n} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ & \vdots & & & \vdots & \\ \frac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_n^{(k)}}{\partial \delta_n} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \hline \frac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_2^{(k)}}{\partial \delta_n} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ & \vdots & & & \vdots & \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_n^{(k)}}{\partial \delta_n} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}$$

In the above equation, bus 1 is assumed to be the slack bus. The Jacobian matrix gives the linearized **relationship** between small changes in voltages angle $\boldsymbol{\delta_i^{(k)}}$ and voltage magnitude $\Delta|\boldsymbol{V_i^{(k)}}|$ with small changes in real and reactive power $\boldsymbol{\partial P_i^{(k)}}$ and $\boldsymbol{\partial Q_i^{(k)}}$.

Elements for the Jacobian matrix are the partial derivatives of (1-52) and (1-53), evaluated at $\Delta\delta_i^{(k)}$ and $\Delta|V_i^{(k)}|$. In short form, It can be written as

$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 J_2 \\ J_3 J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$	(1-54)

For voltage-controlled buses, the voltage magnitude are known. Therefore, if m buses of the system are voltage controlled, m equations involving ΔQ and $\Delta|V|$ and the corresponding columns of the Jacobian matrix are eliminated. accordingly, there are $n-1$ real power constraints and $n-1-m$ reactive power constraints, and the Jacobian matrix is of order $(\boldsymbol{2n-2-m}) \times (\boldsymbol{2n-2-m})$. $\boldsymbol{J_1}$ is of the order of $(n-1)\times(n-1)$. J_2 is of the order $(n-1)\times(n-1-m)$. J_3 is of the order of $(n-1-m)\times(n-1)$, and J_4 is of the order of $(n-1-m)\times(n-1-m)$.

The diagonal and the off-diagonal elements of J_1 are

$$\frac{\partial P_i}{\partial \delta_i}=\sum_{j\neq i}|V_i|\,|V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j) \qquad (1\text{-}55)$$

$$\frac{\partial P_i}{\partial \delta_j}=-|V_i||V_j||Y_{ij}|\sin\left(\theta_{ij}-\delta_i+\delta_j\right) \qquad j\neq i \qquad (1\text{-}56)$$

The diagonal and the off-diagonal elements of J_2 are

$$\frac{\partial P_i}{\partial |V_i|}=2|V_i||Y_{ii}|\cos\theta_{ii}+\sum_{j\neq i}|V_j|\,|Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) \qquad (1\text{-}57)$$

$$\frac{\partial P_i}{\partial |V_j|}=|V_i||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j)\; j\neq i \qquad (1\text{-}58)$$

The diagonal and the off-diagonal elements of J_3 are

$$\frac{\partial Q_i}{\partial \delta_i}=\sum_{j\neq i}|V_i||V_j|\,|Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) \qquad (1\text{-}59)$$

$$\frac{\partial Q_i}{\partial \delta_j}=-|V_i||V_j||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j)\; j\neq i \qquad (1\text{-}60)$$

The diagonal and the off-diagonal elements of J_4 are

$$\frac{\partial Q_i}{\partial |V_i|}=-2|V_i||Y_{ii}|\sin\theta_{ij}-\sum_{j\neq i}|V_j||Y_{ij}|\,sin(\theta_{ij}-\delta_i+\delta_j) \qquad (1\text{-}61)$$

$$\frac{\partial Q_i}{\partial |V_j|}=-|V_i||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j) \qquad j\neq i \qquad (1\text{-}62)$$

The term $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the difference between the scheduled and calculated values, known as the power residuals, given by

$$\Delta P_i^{(k)}=P_i^{sch}-P_i^{(k)} \qquad (1\text{-}63)$$

$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)}$	(1-64)

The new estimates for bus voltages are

$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$	(1-65)

$\lvert V_i^{(k+1)} \rvert = \lvert V_i^{(k)} \rvert + \Delta \lvert V_i^{(k)} \rvert$	(1-66)

The procedure for power flow solution by the newton-Raphson method is as follows:

1. For load buses, where P_i^{sch} and $Q_i^{sch}$are specified, voltage magnitudes and phase angles are equal to the slack bus value, or 1.0 and 0.0, i.e., $|V_i^{(0)}| = 1.0$ and $\boldsymbol{\delta}_i^{(0)} = 0.0$. For voltage-regulated buses, where $|V_i|$and P_i^{sch} are specified, phase angles are set equal to the slack bus angle, or 0, i.e., $\delta_i^{(0)} = 0$.
2. For load buses, $P_i^{(k)}$ and $Q_i^{(k)}$are calculated from (1-52) and (1-53)and $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are calculated from (1-63) and (1-64)
3. For voltage –controlled buses, $P_i^{(k)}$ and $\Delta P_i^{(k)}$are calculated from (1-52) and (1-63), respectively.
4. The elements of the Jacobian matrix (J_1,J_2,J_3 and J_4) are calculated from (1-55)-(1-62).
5. The new voltage magnitudes and phase angles are computed from (1-65) and (1-66).
6. The process is continued until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy, i.e.,

$\lvert \Delta P_i^{(k)} \rvert \leq \varepsilon$ $\lvert \Delta Q_i^{(k)} \rvert \leq \varepsilon$	(1-67)

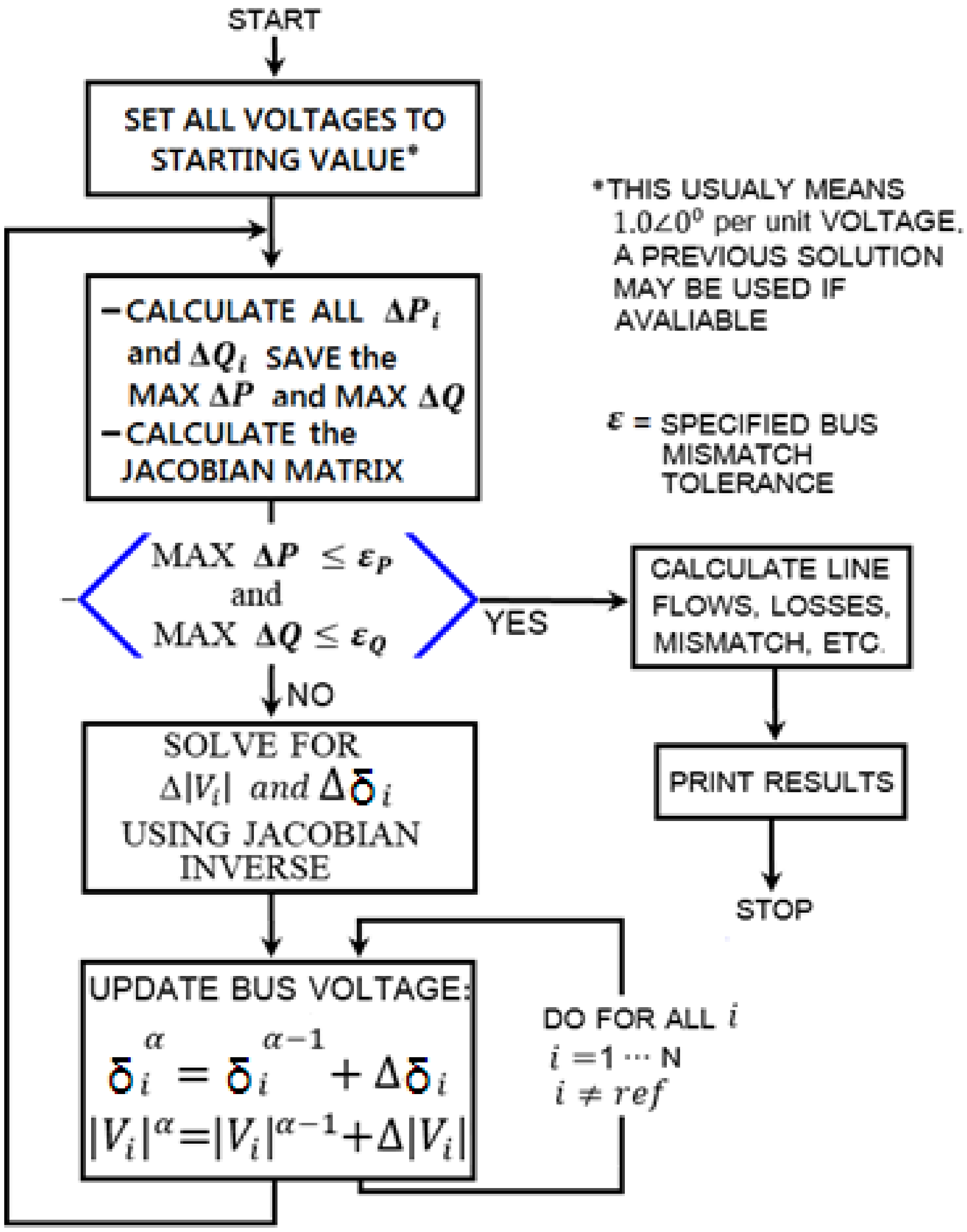

FIG. 1-7 Newton-Raphson power-flow solution

The power flow solution by the newton-Raphson method is demonstrated in the following example.

Example 1-10

Obtain the power flow solution by Newton-Rahpson method for the system of Example 1-8

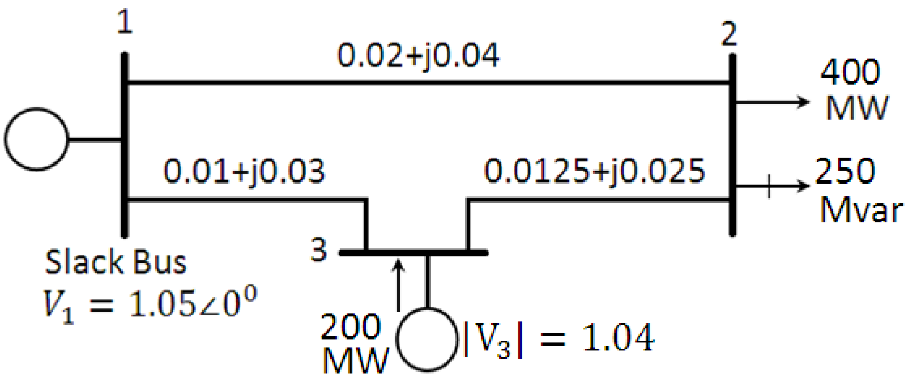

Figure (1-12) One-line diagram of Example 1-8 (impedances in pu on 100 MVA base)

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$, $y_{23} = 16 - j32$. The admittances are on the network shown in Figure 1-12b.

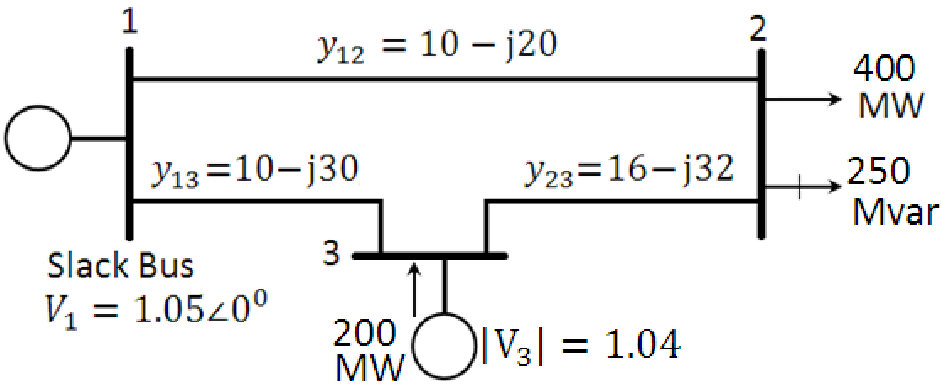

Figure (1-12b) One-line diagram of Example 1-7 (admittances in pu on 100 MVA base)

This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ & -10 + j30\ 16 + j3226 - j62 & \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in **radian** yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.034431.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.10735.7709\angle 2.0344 \\ 31.62278\angle 1.8925 & 22.36068\angle 2.034467.23095\angle -1.1737 \end{bmatrix}$$

From (1-52) and (1-53),

$P_i = \sum_{j=1}^{N} \lvert V_i \rvert \lvert V_j \rvert \lvert Y_{ij} \rvert \cos(\theta_{ij} - \delta_i + \delta_j)$	(1-52)

$$Q_i = -\sum_{j=1}^{N} \lvert V_i\rvert \lvert V_j\rvert \lvert Y_{ij}\rvert \sin(\theta_{ij} - \delta_i + \delta_j)$$	(1-53)

the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33}$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to δ_2, δ_3 and $|V_2|$.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per unit are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5\,pu$$

$$P_3^{sch} = \frac{200}{100} = 2.0\,pu$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$ and $\delta_2^{(0)} = 0.0$. and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (1-63) and (1-64)

$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)}$	(1-63)

$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)}$	(1-64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Evaluation the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.86000 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$\Delta\delta_2^{(0)}$=-0.045263 $\delta_2^{(1)}$=0+(-0.045263)=$-$ 0.045263

$\Delta\delta_3^{(0)}$=-0.007718 $\delta_3^{(1)}$=0+(-0.007718)= $-$0.007718

$\Delta|V_2^{(0)}|$=-0.026548 $|V_2^{(1)}|$=1+(-0.026548) = 0.97345

Voltage phase angles are in radians. For the second iteration, we have

$$\begin{bmatrix}-0.099218\\0.021715\\-0.050914\end{bmatrix}=\begin{bmatrix}51.724675 & -31.765618 & 21.302567\\-32.981642 & 65.656383 & -15.379086\\-28.538577 & 17.402838 & 48.103589\end{bmatrix}\begin{bmatrix}\Delta\delta_2^{(1)}\\\Delta\delta_3^{(1)}\\\Delta|V_2^{(1)}|\end{bmatrix}$$

and

$\Delta\delta_2^{(1)}$=-0.001795 $\delta_2^{(2)}$=0=-0.045263+(-0.001795)=$-$ 0.04706

$\Delta\delta_3^{(1)}$=-0.000985 $\delta_3^{(2)}$=$-$0.007718 +(-0.000985)= $-$0.00870

$\Delta|V_2^{(1)}|$=-0.001767 $|V_2^{(2)}|$=0.973451 +(-0.001767) = 0.971684

For the third iteration, we have

$$\begin{bmatrix}-0.000216\\0.000038\\-0.000143\end{bmatrix}=\begin{bmatrix}51.596701 & -31.693866 & 21.147447\\-32.933865 & 65.597585 & -15.351628\\-28.548205 & 17.396932 & 47.954870\end{bmatrix}\begin{bmatrix}\Delta\delta_2^{(2)}\\\Delta\delta_3^{(2)}\\\Delta|V_2^{(2)}|\end{bmatrix}$$

and

$\Delta\delta_2^{(2)}$=-0.000038 $\delta_2^{(3)}$=0=-0.047058 +(-0.0000038)=$-$ 0.04706

$\Delta\delta_3^{(2)}$=-0.000985 $\delta_3^{(3)}$=$-$0.008703 +(-0.0000024)= $-$0.008705

$\Delta|V_2^{(2)}|$=-0.001767 $|V_2^{(3)}|$=0.971684 +(-0.0000044) = 0.97168

The solution converges in 3 iterations with a maximum power mismatch of 2.5$\times 10^{-4}$ with $V_2 = 0.97168\angle 2.696^0$and $V_3 = 1.04\angle{-0.4988^0}$. from (1-52) and (1-53), the expressions for reactive power at bus 3 and the slack bus real and reactive power are

$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|sin\theta_{33}$$

$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3||Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3)$$

$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3||Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3)$$

Upon substituting, we have

$$Q_3 = 1.4617 \text{ pu}$$
$$P_1 = 2.1842 \text{ pu}$$
$$Q_1 = 1.4085 \text{ pu}$$

Finally the line flow are calculated in the same manner as the line flow calculated in Gauss-Seidel method described in Example 1-7, and the power flow diagram is as shown in Figure 1-13.

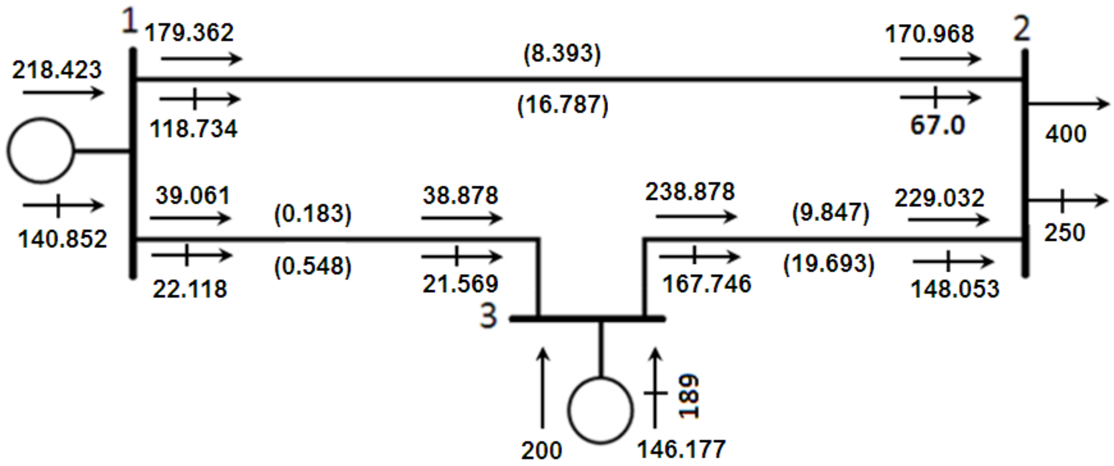

Figure (1-13) Power flow diagram of Example 1-8 (powers in MW and MVAr)